

Abelian realization of phenomenological two-zero neutrino textures

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Abstract

In an attempt at explaining the observed neutrino mass-squared differences and leptonic mixing, lepton mass matrices with zero textures have been widely studied. In the weak basis where the charged lepton mass matrix is diagonal, various neutrino mass matrices with two zeros have been shown to be consistent with the current experimental data. Using the canonical and Smith normal form methods, we construct the minimal Abelian symmetry realizations of these phenomenological two-zero neutrino textures. The implementation of these symmetries in the context of the seesaw mechanism for Majorana neutrino masses is also discussed.

1. Introduction

In the last two decades, neutrino oscillation experiments have firmly established the existence of neutrino masses and lepton mixing. However, there remain several questions to be answered. From experiments we do not know whether neutrinos are Dirac or Majorana particles, and whether CP is violated or not in the lepton sector as it is for quarks (for recent reviews, see e.g. Refs. [1, 2, 3]). In the lack of a convincing theory to explain the origin of neutrino masses and mixing, various approaches to the flavour puzzle have been pursued. In particular, the imposition of some texture zeros in the neutrino mass matrix allows to reduce the number of free parameters, and to establish certain relations between the flavour mixing angles and mass ratios that could be testable.

A common theoretical issue with mass (or coupling) matrices containing vanishing entries is the origin of such texture zeros. It is possible to enforce texture zeros in arbitrary entries of the fermion mass matrices by means of discrete Abelian symmetries, e.g., the cyclic groups Z_n [4, 5]. Yet the general methods commonly used to obtain such patterns do not necessarily lead to their simplest realization, i.e. with the smallest discrete Abelian group and number of Higgs scalars.

In the basis where the charged lepton mass matrix is diagonal, the two-zero textures for the effective neutrino mass matrix are phenomenological Ansätze, first studied by Frampton, Glashow and Marfatia (FGM) in Ref. [6]. It turns out that only a subset of them is presently compatible with the neutrino oscillation data [7, 8]. The aim of this work is to construct the minimal Abelian symmetry

realizations of these phenomenological two-zero neutrino textures, and to study their implementation in extensions of the standard model (SM) based on the seesaw mechanism for the neutrino masses. In our search, we shall combine the canonical and Smith normal form (SNF) methods, which have proved to be very successful in this context [9, 10, 11, 12, 13].

2. Two methods and their complementarity in model building

There are two main approaches that can be used to study Abelian symmetries in the Lagrangian: the canonical method (see Refs. [10, 12] and references therein) and the SNF method [11, 13]. In this section, we shall briefly review these two methods and show their complementarity in studying neutrino mass matrices with texture zeros.

In the canonical method, when dealing with Abelian symmetries, we represent the generators of the symmetry group by diagonal phase matrices, i.e. $\mathcal{S} = \text{diag}(e^{i\alpha}, e^{i\beta}, \dots)$ for each set of flavours. Consider, for example, the Yukawa-type interaction,

$$\bar{L}_\alpha Y_{\alpha\beta}^a R_\beta \Phi_a, \quad (1)$$

with $\alpha, \beta = 1, \dots, n_f$ and $a = 1, \dots, n_h$; L, R , denote the left-handed and right-handed fermion fields, respectively; Φ are the Higgs fields. To study the symmetries of this interaction, we impose its invariance under the field transformations

$$L \rightarrow \mathcal{S}_L L, R \rightarrow \mathcal{S}_R R, \Phi \rightarrow \mathcal{S}_\Phi \Phi. \quad (2)$$

This leads to the symmetry relations

$$\mathcal{S}_L^\dagger Y^a \mathcal{S}_R (\mathcal{S}_\Phi)_a = Y^a, \quad (3)$$

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where here, and henceforth, no summation over a is assumed.

The above relations can be simplified by going to the Hermitian combinations $H_L^a = Y^a Y^{a\dagger}$ and $H_R^a = Y^{a\dagger} Y^a$, which have the symmetry relations

$$\mathcal{S}_L^\dagger H_L^a \mathcal{S}_L = H_L^a, \quad \mathcal{S}_R^\dagger H_R^a \mathcal{S}_R = H_R^a. \quad (4)$$

From Eqs. (3) and (4), one can then determine the allowed textures of Y^a , for a given n_f and independently of n_h [12]. Note however that, in this approach, the determination of the symmetry charges is a combinatorial problem which can be very tedious.

In the Smith normal form method, a slightly different approach is followed. The method uses the fact that if there are n_F flavour fields (in our example, $n_F = 2n_f + n_h$), the Lagrangian would have an Abelian symmetry $[U(1)]^{n_F}$ in the absence of phase sensitive terms. A term where a given field appears only in combinations with its conjugated one, for example, $\Phi_a^\dagger \Phi_a$, is not sensitive to the phase of that field. The phase sensitive interactions will constrain the $U(1)$ groups by establishing correlations between different groups or breaking them completely.

Let us take a simple example of two complex scalar fields ϕ_1 and ϕ_2 . The phase insensitive Lagrangian is invariant under $U(1) \times U(1)$, under which the fields transform as $\phi_1 \rightarrow (e^{i\alpha_1}, 1) \phi_1$ and $\phi_2 \rightarrow (1, e^{i\alpha_2}) \phi_2$. In the presence of the interaction term $\phi_1^\dagger \phi_2$, we are no longer able to rotate ϕ_1 and ϕ_2 arbitrarily. Both fields need to be rotated with the same phase α . Thus our initial symmetry is broken down to a single continuous Abelian group, i.e. $U(1) \times U(1) \rightarrow Z_1 \times U(1) = U(1)$, with $\phi_{1,2} \rightarrow e^{i\alpha} \phi_{1,2}$ as the symmetry transformation. If, on the other hand, the term ϕ_1^2 is added, the rephasing symmetry associated with ϕ_2 will remain unchanged, but the symmetry of ϕ_1 is broken to Z_2 . We then get $U(1) \times U(1) \rightarrow Z_2 \times U(1)$, with $\phi_1 \rightarrow (-1, 1) \phi_1$ and $\phi_2 \rightarrow (1, e^{i\alpha}) \phi_2$ as the symmetry transformation.

The idea of the SNF method is to deal with the symmetry breaking in a generic way. To see how this is done we start by building a vector containing all the fields. For the Yukawa interaction term of Eq. (1), these are

$$(\Phi_a, L_\alpha, R_\beta). \quad (5)$$

The sequence of the fields is irrelevant, but once an ordering is chosen it must be kept until the end of the calculation. The steps for applying the method are the following:

(i) For each phase sensitive interaction, we build a vector of the form of Eq. (5) where the entry j , associated with a particular flavour field, is the number of fields minus the number of conjugated ones.

For example, considering the terms $\bar{L}_1 R_2 \Phi_1$ and $\bar{L}_3 R_2 \Phi_2$, one writes

$$\underbrace{(1, 0, \dots)}_{\Phi_a}, \underbrace{(-1, 0, 0, \dots)}_{L_\alpha}, \underbrace{(0, 1, \dots)}_{R_\beta} \quad (6)$$

for the first term, and

$$(0, 1, \dots, 0, 0, -1, \dots, 0, 1, \dots) \quad (7)$$

for the second one.

(ii) With the k phase sensitive terms, we construct a $k \times n_F$ matrix $D = \{d_{ij}\}$, where each row contains one of the vectors built in (i). Since the Lagrangian must be invariant, the system of coupled equations $d_{ij} \alpha_j = 2\pi n_i$, with $n_i \in \mathbb{Z}$, has to be satisfied.

(iii) We bring the matrix D to its Smith normal form D_{SNF} , defined as

$$D_{SNF} = \text{diag}(d_1, d_2, \dots, d_r, 0, \dots, 0), \quad (8)$$

with positive integers d_i such that d_i is a divisor of d_{i+1} and $r = \text{rank}(D)$. Note that the matrix D_{SNF} is rectangular when $k \neq n_F$, so that Eq. (8) means that everything else away from the square block is also filled with zeros. For any integer value matrix D there is a unique D_{SNF} associated with it and related by $D = R D_{SNF} C$. The matrices R and C encode the operations (addition, sign flip and permutation) on the rows and columns, respectively.

(iv) At this point, the system of equations has been transformed into a system of uncoupled equations $d_j \tilde{\alpha}_j = 2\pi \tilde{n}_j$, with $j = 1, \dots, r$, $\tilde{\alpha}_j = (C\alpha)_j$ and $\tilde{n}_i = (R^{-1}n)_i$.¹ For $d_j \neq 0$ each equation corresponds to a Z_{d_j} group, while for $d_j = 0$ it corresponds to a $U(1)$ group. Thus the symmetry of the Lagrangian has been broken down to

$$[U(1)]^{n_F} \rightarrow Z_{d_1} \times \dots \times Z_{d_r} \times [U(1)]^{n_F - r}. \quad (9)$$

The original independent phases are now written as

$$\alpha_j = \left(\frac{2\pi}{d_1} (C^{-1})_{j1}, \dots, \frac{2\pi}{d_r} (C^{-1})_{jr}, \beta_{r+1}, \dots, \beta_{n_F} \right). \quad (10)$$

This simple procedure allows us to extract important information from the presence of discrete and continuous symmetries in the Lagrangian. As elegantly shown by Ivanov and Nishi [13], general conditions for the possible model implementations without accidental $U(1)$'s can be found. This approach has advantages over the canonical one [12], since the latter needs an explicit construction to fully classify these models. Still, we remark that both methods have interesting features that can be complementary from a model building viewpoint. Note that, while in Eq. (10) the discrete phases are predicted by the SNF method, the phases of the continuous groups are not. Those can be easily obtained by the canonical method. It is also very common in model building the implementation of specific zero-textures through the use of symmetries.

¹Note that, since the matrices R and C are invertible with $|\det| = 1$, we do not loose or generate any new solution when switching from the system with D to the system with D_{SNF} . The symmetries are the same, and the solutions are in one-to-one correspondence.

However, these zero-textures are in general present in the mass terms and not in the interaction ones. To perform such a bottom-up approach, it is useful to use the information on the allowed combinations of textures, which can be easily constructed within the canonical method.

In this work, we shall use a bottom-up approach to answer the following question: Given a model where the neutrino mass matrix exhibits a two-zero texture in the flavour basis, what are the minimal Abelian symmetries that can be implemented to obtain such a pattern? In the next sections we shall address this question using the canonical and SNF methods at different stages of the problem.

3. Textures from Abelian symmetries in the leptonic sector

The origin of Majorana neutrino masses can be explained through the introduction of the unique dimension-five operator compatible with the SM gauge group. The leptonic interaction Lagrangian can be written as

$$-\mathcal{L}_{\text{int}} = \Pi_{\alpha\beta}^a \overline{\ell_{\alpha L}} \phi_a e_{\beta R} + \frac{K_{\alpha\beta}^{ab}}{2\Lambda} \left(\overline{\ell_{\alpha L}} \tilde{\phi}_a \right) \left(\tilde{\phi}_b^T \ell_{\beta L}^c \right), \quad (11)$$

where Λ is an effective energy scale, ℓ_L denotes the left-handed lepton doublet fields and e_R are the right-handed charged-lepton singlets, $\tilde{\phi} = i\sigma_2 \phi^*$, and we allow for the possibility of extra Higgs doublet fields ϕ_a ; $a, b = 1, \dots, n_h$.

In the presence of flavour symmetries, two situations may occur: either the high-dimensional operator is invariant under the full flavour symmetry or it breaks the full flavour symmetry completely (or to a subgroup). The latter case may arise from the ultraviolet (UV) completion of the model. The flavour group at the UV level can be broken spontaneously through some additional scalar fields (flavons), or it can even be broken explicitly by dimension three (or less) operators. In what follows we shall focus only on the former situation, i.e. when the dimension-five Weinberg operator is invariant under the full flavour symmetry. In this case, a symmetry pattern can be defined and the study of textures can be done in a model independent way.

Since we are only interested in the study of Abelian symmetries, following the canonical approach we define the field transformations $\ell_L \rightarrow \mathcal{S}_L \ell_L$, $e_R \rightarrow \mathcal{S}_R e_R$, and $\Phi \rightarrow \mathcal{S}_\Phi \Phi$, written in the basis where all the transformations are diagonal unitary matrices, i.e.

$$\begin{aligned} \mathcal{S}_L &= \text{diag} (e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}), \\ \mathcal{S}_R &= \text{diag} (e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3}), \\ \mathcal{S}_\Phi &= \text{diag} (e^{i\theta_1}, e^{i\theta_2}, \dots, e^{i\theta_N}). \end{aligned} \quad (12)$$

These field transformations lead to the following symmetry relations

$$\mathcal{S}_L^\dagger \Pi^a \mathcal{S}_R e^{i\theta_a} = \Pi^a, \quad \mathcal{S}_L^\dagger K^{ab} \mathcal{S}_L^* e^{-i(\theta_a + \theta_b)} = K^{ab}. \quad (13)$$

The textures for the Yukawa coupling matrices Π^a are identical to the quark sector [12], while for the matrix K^{ab} we should keep only the symmetric textures. Therefore, the allowed textures for K^{ab} are

$$P^T \{A_1, A_2, A_3, A_7, A_{12}\} P \quad (14)$$

for $P = \mathbb{1}$, P_{13} or P_{23} , and

$$[P' \{A_{13}, A_{15}\} P]_{\text{sym}} \quad (15)$$

for any P and P' , with P and P' denoting the 3×3 permutation matrices,

$$\begin{aligned} \mathbb{1} &= \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}, \quad P_{12} = \begin{pmatrix} & 1 & \\ 1 & & \\ & & 1 \end{pmatrix}, \\ P_{13} &= \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}, \quad P_{23} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}, \\ P_{123} &= \begin{pmatrix} & & 1 \\ 1 & & \\ & 1 & \end{pmatrix}, \quad P_{321} = \begin{pmatrix} & 1 & \\ & & 1 \\ 1 & & \end{pmatrix}. \end{aligned} \quad (16)$$

The textures A_i have the explicit patterns

$$\begin{aligned} A_1 &= \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad A_2 = \begin{pmatrix} \times & \times & \\ \times & \times & \\ & & \times \end{pmatrix}, \\ A_3 &= \begin{pmatrix} & \times & \\ & \times & \\ \times & \times & \end{pmatrix}, \quad A_7 = \begin{pmatrix} \times & \times & \\ \times & \times & \\ & & \end{pmatrix}, \\ A_{12} &= \begin{pmatrix} & & \\ & \times & \\ & & \times \end{pmatrix}, \quad A_{13} = \begin{pmatrix} \times & & \\ & \times & \\ & & \times \end{pmatrix}, \\ A_{15} &= \begin{pmatrix} & & \\ \times & & \\ & & \times \end{pmatrix}, \end{aligned} \quad (17)$$

where \times denotes a nonzero matrix element. Since \mathcal{S}_L acts both on left and right of K^{ab} , we can group the possible textures into three classes:

- (1) $\mathcal{S}_L = \mathbb{1}$ is completely degenerate. The only possible texture that can be implemented is A_1 .
- (2) $\mathcal{S}_L = P^T \text{diag} (1, 1, e^{i\alpha}) P$, i.e. there is a two-fold degeneracy. This class can be seen as three subclasses, one for each matrix $P = \{\mathbb{1}, P_{13}, P_{23}\}$. The allowed textures are $P^T \{A_2, A_3, A_7, A_{12}\} P$.
- (3) $\mathcal{S}_L = \text{diag} (1, e^{i\alpha_1}, e^{i\alpha_2})$ is nondegenerate. For any P and P' , the allowed textures are $[P' \{A_{12}, A_{13}, A_{15}\} P]_{\text{sym}}$.

In the next section, we make use of the above classification in order to reconstruct the FGM two-zero textures in terms of the textures allowed by the Abelian symmetries.

4. Decomposing the FGM two-zero textures

The neutrino mass matrix is a symmetric matrix with six independent complex entries. There are fifteen textures with two independent texture zeros, usually classified into six categories $(\mathbf{A}_{1,2}, \mathbf{B}_{1,2,3,4}, \mathbf{C}, \mathbf{D}_{1,2}, \mathbf{E}_{1,2,3}, \mathbf{F}_{1,2,3})$ [6]. While this classification may be advantageous for phenomenological studies, from the symmetry viewpoint it is more convenient to group them in a slightly different way [14]:

$$\begin{aligned}\tilde{\mathbf{A}} &\equiv \{\mathbf{A}_1, \mathbf{A}_2 = P_{23}\mathbf{A}_1P_{23}, \mathbf{B}_3 = P_{12}\mathbf{A}_1P_{12}, \\ &\quad \mathbf{B}_4 = P_{321}\mathbf{A}_1P_{123}, \mathbf{D}_1 = P_{123}\mathbf{A}_1P_{321}, \\ &\quad \mathbf{D}_2 = P_{13}\mathbf{A}_1P_{13}\}, \\ \tilde{\mathbf{B}} &\equiv \{\mathbf{B}_1, \mathbf{B}_2 = P_{23}\mathbf{B}_1P_{23}, \mathbf{E}_3 = P_{12}\mathbf{B}_1P_{12}\}, \\ \tilde{\mathbf{C}} &\equiv \{\mathbf{C}, \mathbf{E}_1 = P_{321}\mathbf{C}P_{123}, \mathbf{E}_2 = P_{12}\mathbf{C}P_{12}\}, \\ \tilde{\mathbf{F}} &\equiv \{\mathbf{F}_1, \mathbf{F}_2 = P_{12}\mathbf{F}_1P_{12}, \mathbf{F}_3 = P_{13}\mathbf{F}_1P_{13}\},\end{aligned}\quad (18)$$

where

$$\begin{aligned}\mathbf{A}_1 &: \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad \mathbf{B}_1: \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}, \\ \mathbf{C} &: \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}, \quad \mathbf{F}_1: \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}.\end{aligned}\quad (19)$$

In the flavour basis, the only two-zero neutrino textures that can be obtained with a single scalar Higgs doublet are the ones in class $\tilde{\mathbf{F}}$. This is a direct consequence of Eqs. (17), since no other two-zero texture is in the set of allowed textures. This class is however phenomenologically excluded. Therefore, in order to implement the phenomenologically viable two-zero textures \mathbf{A}_i , \mathbf{B}_i and \mathbf{C} , an extended scalar sector with $n_h \geq 2$ is needed. Following a path similar to that of Ref. [12], we shall use a bottom-up approach in order to find the possible ways of implementing the above two-zero textures.

To illustrate our approach, we consider class $\tilde{\mathbf{B}}$ and choose the texture \mathbf{B}_1 . The first step is to decompose the zero-texture \mathbf{B}_1 in the largest possible set of textures, keeping in mind that the effective neutrino mass matrix should be symmetric. We obtain the decomposition

$$\begin{aligned}\mathbf{B}_1 &= \begin{pmatrix} \times & \times & \\ \times & & \times \\ & \times & \times \end{pmatrix} + \begin{pmatrix} \times & & \\ & \times & \times \\ & & \times \end{pmatrix} + \begin{pmatrix} \times & & \\ & \times & \\ & & \times \end{pmatrix} + \begin{pmatrix} & \times & \\ & & \times \\ & & \times \end{pmatrix} \\ &= P_{321}A_{15}P_{13} \oplus A_{15}P_{23} \oplus P_{13}A_{12}P_{13} \oplus A_{12}.\end{aligned}\quad (20)$$

The maximal number of different textures that build \mathbf{B}_1 is four. Any model with more than four interaction terms will have necessarily repeated textures.

The next step is to reduce the above decomposition from four to three textures. There are six possibilities. Summing the first and second textures we get the decomposition

$$\begin{aligned}\mathbf{B}_1 &= \begin{pmatrix} \times & \times & \\ \times & & \times \\ & \times & \times \end{pmatrix} + \begin{pmatrix} \times & & \\ & \times & \\ & & \times \end{pmatrix} + \begin{pmatrix} & \times & \\ & & \times \\ & & \times \end{pmatrix} \\ &= P_{23}A_3P_{23} \oplus P_{13}A_{12}P_{13} \oplus A_{12}.\end{aligned}\quad (21)$$

The first texture is not compatible with the other two. Indeed, although the three textures belong to class (2), the first one belongs to the subclass with $P = P_{23}$, while the second and third textures are in the subclasses with $P = P_{13}$ and $P = \mathbb{1}$, respectively.

Summing the first and third textures we get the decomposition

$$\mathbf{B}_1 = \begin{pmatrix} \times & \times & \\ \times & & \times \\ & \times & \times \end{pmatrix} + \begin{pmatrix} \times & & \\ & \times & \times \\ & & \times \end{pmatrix} + \begin{pmatrix} & \times & \\ & & \times \\ & & \times \end{pmatrix}.\quad (22)$$

In this case, the first texture does not exist in the set of textures allowed by the Abelian symmetries.

Summing the first and fourth textures we obtain

$$\begin{aligned}\mathbf{B}_1 &= \begin{pmatrix} \times & \times & \\ \times & & \times \\ & \times & \times \end{pmatrix} + \begin{pmatrix} \times & & \\ & \times & \times \\ & & \times \end{pmatrix} + \begin{pmatrix} \times & & \\ & \times & \\ & & \times \end{pmatrix} \\ &= A_{13}P_{12} \oplus A_{15}P_{23} \oplus P_{13}A_{12}P_{13},\end{aligned}\quad (23)$$

which are compatible textures belonging to class (3). We can continue the above procedure, obtaining at the end two more implementations with three textures, namely, $A_{13}P_{23} \oplus P_{321}A_{15}P_{13} \oplus A_{12}$, and $P_{321}A_{15}P_{13} \oplus A_{15}P_{23} \oplus P_{12}A_{15}P_{12}$.

Similarly, matrices with only two textures can be constructed. We need to look just at the allowed cases with three textures. It turns out that only in the first case there exists a viable implementation of two textures, which is given by the decomposition $A_{13}P_{12} \oplus A_{13}P_{23}$.

Below we summarize the allowed texture decompositions for the three classes $\tilde{\mathbf{A}}$, $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{C}}$:

$\tilde{\mathbf{A}}$	Class
(a)	$P_{12}A_{15}P_{321} \oplus A_{15}P_{23} \oplus P_{23}A_{12}P_{23} \oplus A_{12},$
(b)	$A_{13}P_{13} \oplus A_{15}P_{23} \oplus A_{12},$
(c)	$P_{12}A_{15}P_{321} \oplus A_{15}P_{23} \oplus A_{15}.$

(24)

$\tilde{\mathbf{B}}$ Class

- (a) $P_{321}A_{15}P_{13} \oplus A_{15}P_{23} \oplus P_{13}A_{12}P_{13} \oplus A_{12},$
- (b) $A_{13}P_{12} \oplus A_{15}P_{23} \oplus P_{13}A_{12}P_{13},$ (25)
- (c) $P_{321}A_{15}P_{13} \oplus A_{15}P_{23} \oplus P_{12}A_{15}P_{12},$
- (d) $A_{13}P_{12} \oplus A_{13}P_{23}.$

$\tilde{\mathbf{C}}$ Class

- (a) $P_{321}A_{15}P_{13} \oplus P_{123}A_{15}P_{12} \oplus A_{15}P_{23} \oplus P_{13}A_{12}P_{13},$
- (b) $A_{13}P_{23} \oplus P_{321}A_{15}P_{13} \oplus P_{123}A_{15}P_{12}.$ (26)

Next we shall study the realization of the above decompositions through the introduction of Abelian symmetries in the context of 2HDM.

5. Implementation of FGM two-zero textures in 2HDM

In our previous analysis, we have made use of the canonical method to obtain the possible textures that implement the FGM two-zero textures. In this section, we shall use the SNF method in order to find the corresponding Abelian symmetries. The vector containing the relevant fields of the leptonic sector, in the presence of the $d = 5$ effective operator, is defined as $(\phi_a, \ell_{\alpha L}, e_{\beta R})$.

The FGM two-zero textures are written in the basis where the charged leptons are diagonal. Inserting this requirement into the matrix D and performing some simple operations on rows and columns (see steps (ii) and (iii) for the SNF method described in section 2), it will lead to a structure of the form

$$D_{k \times (n_h+6)} \sim \left(\begin{array}{c|c} \mathbb{0}_{3 \times (n_h+3)} & \mathbb{1}_3 \\ \hline \mathcal{D}_{(k-3) \times (n_h+3)} & \mathbb{0}_{(k-3) \times 3} \end{array} \right). \quad (27)$$

This implies that, in order to find the symmetries in the leptonic sector, we only need to look at the matrix \mathcal{D} and the associated field vector (ϕ_a, ℓ_α) of the effective operator interactions.²

First we note that there are always $n_h + 3 - r$ continuous Abelian symmetries in our subsystem, as can be seen from Eq. (9). The rank of \mathcal{D} is at most 4, which is the number

²Notice that there are also Abelian symmetries in the charged lepton sector. For example, if only one Higgs doublet is coupled to charged leptons, then $n_F = 7$ and $r = 3$ (three nonzero interactions). This implies the presence of four Abelian continuous symmetries: the global hypercharge and $U(1)_e \times U(1)_\mu \times U(1)_\tau$. Nevertheless, the neutrino interaction terms break the latter symmetry down, so that the flavour symmetry in the whole Lagrangian is dictated by the neutrino textures.

of distinct interactions in the effective term. Therefore, for the FGM two-zero textures there exist $n_h - 1$ continuous Abelian symmetries. Since the global hypercharge $U(1)_Y$ is always present, only two Higgs doublet models (2HDM) may avoid additional continuous symmetries in the leptonic sector. The most dangerous continuous symmetries are the ones present in the scalar potential, since they may lead to (pseudo-) Goldstone bosons. For now, we shall only focus on the leptonic sector in order to find the minimal discrete Abelian realizations of the FGM two-zero textures in 2HDM. We shall then comment on the symmetries of the scalar sector at the end of this section.

Working in a framework with two scalar fields, $\phi_{1,2}$, we can only form three distinct combinations: $(\phi_1)^2$, $(\phi_2)^2$ and $\phi_1\phi_2$. This implies that the three cases labelled (a) in Eqs. (24)-(26) are automatically excluded. To see how the symmetries can be straightforwardly determined using the SNF method, let us consider, for instance, the decomposition (b) in class $\tilde{\mathbf{B}}$. One possibility for the interaction Lagrangian is

$$-\mathcal{L}_{int}^{d=5} = \overline{\ell}_L [A_{13}P_{12}(\phi_1^*)^2 + A_{15}P_{23}(\phi_2^*)^2 + P_{13}A_{12}P_{13}(\phi_1^*\phi_2^*)] \ell_L^c. \quad (28)$$

From this Lagrangian we build the matrix \mathcal{D} ,

$$\mathcal{D} = \begin{pmatrix} -2 & 0 & -1 & -1 & 0 \\ -2 & 0 & 0 & 0 & -2 \\ 0 & -2 & 0 & -1 & -1 \\ -1 & -1 & -2 & 0 & 0 \end{pmatrix}, \quad (29)$$

which has the Smith normal form $\mathcal{D}_{SNF} = \text{diag}(1, 1, 1, 10)$, and leads to the symmetry $Z_{10} \sim Z_2 \times Z_5$. As explained in step (iv) of section 2, one can extract the discrete charges for the flavour symmetry using the information coming from the operations on columns (i.e. from matrix C). We get

$$Z_2 \times Z_5 : \begin{cases} \phi_1 \rightarrow (-1, 1) \phi_1, & \phi_2 \rightarrow (-1, \omega_5^2) \phi_2, \\ \ell_{eL} \rightarrow (1, \omega_5^4) \ell_{eL}, & \ell_{\mu L} \rightarrow (1, \omega_5) \ell_{\mu L}, \\ \ell_{\tau L} \rightarrow (1, 1) \ell_{\tau L}, \end{cases} \quad (30)$$

where $\omega_n = e^{i2\pi/n}$.

Note that the Z_2 group is simply the discrete lepton number that remains after the explicit breaking of $U(1)_L$ by the $d = 5$ effective operator. Since in this implementation we have coupled $(\phi_1^*)^2$ to the first texture, $(\phi_2^*)^2$ to the second one, and $\phi_1^*\phi_2^*$ to the third one, we denote this Higgs combination by $[(1, 1), (2, 2), (1, 2)]$. Checking all the other Higgs combinations, we conclude that none of them has a symmetry implementation.

The same procedure can be used to find the symmetry implementations for all classes. In Table 1 we summarize the texture decompositions that can be implemented, their Higgs combinations and the associated Abelian symmetries. For completeness, we present below the symmetry charges for the other cases given in Table 1. For class \mathbf{A}

Table 1: Two-zero texture decompositions, Higgs combinations and the corresponding Abelian symmetries for classes $\tilde{\mathbf{A}}$, $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{C}}$.

Texture	Texture decomposition	Higgs combination	Symmetry
$\tilde{\mathbf{A}} = \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$	$A_{13}P_{13} \oplus A_{15}P_{23} \oplus A_{12}$ $\begin{pmatrix} & & \times \\ & \times & \\ \times & & \end{pmatrix} + \begin{pmatrix} & & \\ & \times & \\ \times & & \end{pmatrix} + \begin{pmatrix} & & \\ & & \times \\ & \times & \end{pmatrix}$	$[(1, 1), (1, 2), (2, 2)]$	$Z_2 \times U(1)$
$\tilde{\mathbf{B}} = \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$	$A_{13}P_{12} \oplus A_{15}P_{23} \oplus P_{13}A_{12}P_{13}$ $\begin{pmatrix} & \times & \\ \times & & \times \end{pmatrix} + \begin{pmatrix} & & \\ & \times & \\ \times & & \end{pmatrix} + \begin{pmatrix} \times & & \\ & \times & \\ & & \times \end{pmatrix}$ $P_{321}A_{15}P_{13} \oplus A_{15}P_{23} \oplus P_{12}A_{15}P_{12}$ $\begin{pmatrix} \times & & \\ & \times & \\ \times & & \end{pmatrix} + \begin{pmatrix} & & \\ & \times & \\ \times & & \end{pmatrix} + \begin{pmatrix} \times & & \\ & & \times \\ & \times & \end{pmatrix}$	$[(1, 1), (2, 2), (1, 2)]$	$Z_2 \times Z_5$
	$A_{13}P_{12} \oplus A_{13}P_{23}$ $\begin{pmatrix} & \times & \\ \times & & \times \end{pmatrix} + \begin{pmatrix} \times & & \\ & \times & \\ & & \times \end{pmatrix}$	$[(1, 1), (2, 2)]$	$Z_2 \times Z_2 \times Z_3$
$\tilde{\mathbf{C}} = \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$	$A_{13}P_{23} \oplus P_{321}A_{15}P_{13} \oplus P_{123}A_{15}P_{12}$ $\begin{pmatrix} \times & & \\ & \times & \\ \times & & \end{pmatrix} + \begin{pmatrix} \times & & \\ & \times & \\ \times & & \end{pmatrix} + \begin{pmatrix} & & \times \\ \times & & \\ \times & & \end{pmatrix}$	$[(1, 2), (1, 1), (2, 2)]$	$U(1)$

we have

$$Z_2 \times U(1) :$$

$$\begin{cases} \phi_1 \rightarrow (-1, 1) \phi_1, \\ \phi_2 \rightarrow (-1, e^{i\alpha}) \phi_2, \\ \ell_{eL} \rightarrow (1, e^{i\alpha}) \ell_{eL}, \\ \ell_{\mu L} \rightarrow (1, 1) \ell_{\mu L}, \\ \ell_{\tau L} \rightarrow (1, e^{-i\alpha}) \ell_{\tau L}, \end{cases} \quad \supset \quad \begin{cases} \phi_1 \rightarrow (-1, 1) \phi_1, \\ \phi_2 \rightarrow (-1, \omega_5) \phi_2, \\ \ell_{eL} \rightarrow (1, \omega_5) \ell_{eL}, \\ \ell_{\mu L} \rightarrow (1, 1) \ell_{\mu L}, \\ \ell_{\tau L} \rightarrow (1, \omega_5^4) \ell_{\tau L}. \end{cases} \quad (31)$$

While the SNF method points in this case to the existence of a continuous Abelian symmetry $U(1)$, there is a minimal discrete Abelian realization Z_5 that leads to this texture. This symmetry can be found within the canonical method (see Table A.1 in Appendix A).

For the remaining decompositions of class $\tilde{\mathbf{B}}$ we get

$$Z_8 : \begin{cases} \phi_1 \rightarrow \phi_1, & \phi_2 \rightarrow \omega_8^2 \phi_2, \\ \ell_{eL} \rightarrow \omega_8^3 \ell_{eL}, & \ell_{\mu L} \rightarrow \omega_8^5 \ell_{\mu L}, \\ \ell_{\tau L} \rightarrow \omega_8^7 \ell_{\tau L}, \end{cases} \quad (32)$$

and

$$Z_2 \times Z_2 \times Z_3 : \begin{cases} \phi_1 \rightarrow (1, 1, 1) \phi_1, \\ \phi_2 \rightarrow (1, -1, \omega_3) \phi_2, \\ \ell_{eL} \rightarrow (-1, -1, \omega_3^2) \ell_{eL}, \\ \ell_{\mu L} \rightarrow (-1, -1, \omega_3) \ell_{\mu L}, \\ \ell_{\tau L} \rightarrow (-1, -1, 1) \ell_{\tau L}, \end{cases} \quad (33)$$

where the group Z_6 , coming from the Smith normal form, has been decomposed as $Z_2 \times Z_3$.

Finally, for class $\tilde{\mathbf{C}}$ we obtain³

$$U(1) : \quad Z_8 :$$

$$\begin{cases} \phi_1 \rightarrow e^{-i\alpha/2} \phi_1, \\ \phi_2 \rightarrow e^{i\alpha/2} \phi_2, \\ \ell_{eL} \rightarrow \ell_{eL}, \\ \ell_{\mu L} \rightarrow e^{i\alpha} \ell_{\mu L}, \\ \ell_{\tau L} \rightarrow e^{-i\alpha} \ell_{\tau L}, \end{cases} \quad \supset \quad \begin{cases} \phi_1 \rightarrow \phi_1, \\ \phi_2 \rightarrow \omega_8^2 \phi_2, \\ \ell_{eL} \rightarrow \omega_8^7 \ell_{eL}, \\ \ell_{\mu L} \rightarrow \omega_8 \ell_{\mu L}, \\ \ell_{\tau L} \rightarrow \omega_8^5 \ell_{\tau L}, \end{cases} \quad (34)$$

where the minimal discrete Abelian realization Z_8 that leads to this texture can be easily found resorting to the canonical method (cf. Table A.1 in Appendix A).

³The Higgs field transformation for the discrete symmetry Z_8 is brought to the form given in Eq. (34) by means of the global phase transformation $\phi_a \rightarrow \omega_8 \phi_a$ and $\ell_{\alpha L} \rightarrow \omega_8^{-1} \ell_{\alpha L}$.

Table 2: Minimal symmetry groups for the 2HDM effective scalar sector.

Texture	Symmetry of the fields	Symmetry of the potential
$\tilde{\mathbf{A}}$	Z_5	
$\tilde{\mathbf{B}}$	Z_5 Z_4 $Z_2 \times Z_3$	$U(1)$
$\tilde{\mathbf{C}}$	Z_4	

The symmetries obtained should not be understood loosely. In the case of discrete symmetries, these are the minimal symmetries that lead to the required effective neutrino textures. For continuous symmetries, our construction implies that, even though there could be some discrete symmetry implementing such textures, they always lead to a continuous symmetry in the effective Lagrangian. Notice however that these are symmetries at the effective level; they are not necessarily present at the UV level. Actually, depending on the UV completion, we may only need subgroups of the effective flavour group.

We end this section by analysing the scalar sector. At the effective level we have only two Higgs doublets, transforming as $\phi_1 \rightarrow \phi_1, \phi_2 \rightarrow \omega_n \phi_2$ under the flavour symmetry. We summarize in Table 2 the minimal discrete groups present in the scalar potential. Since the scalar potential contains only two Higgs doublets, the largest discrete Abelian symmetry is Z_2 [11]. A larger group would lead to a continuous accidental $U(1)$. Therefore, in order to avoid the unwanted (pseudo-) Goldstone bosons, an UV completion of these models is needed.

In principle, continuous symmetries in the scalar sector are dangerous only when they are spontaneously broken. If the 2HDM scalar potential produces an inert-like vacuum expectation value (vev) alignment $\langle \phi_1 \rangle \neq 0, \langle \phi_2 \rangle = 0$, then the symmetry remains unbroken. But, in this case, some terms in the texture decomposition will not contribute to the neutrino mass matrix and the desired texture cannot be constructed. This argument validates the point that we should avoid scalar $U(1)$'s.

6. Symmetry realization of the FGM two-zero textures in a seesaw framework

Perhaps the simplest UV completions of the effective models previously discussed are those based on the seesaw models for the neutrino masses. Next we present the implementations of the minimal Abelian symmetries for the two-zero neutrino textures in the context of the type I and type II seesaw mechanisms.

6.1. Type-II seesaw realization

In the type II seesaw framework, $SU(2)_L$ triplet scalars Δ_k with hypercharge $Y = 1$,

$$\Delta_k = \begin{pmatrix} \Delta_k^0 & -\frac{\Delta_k^+}{\sqrt{2}} \\ -\frac{\Delta_k^+}{\sqrt{2}} & \Delta_k^{++} \end{pmatrix}, \quad (35)$$

are added to the SM particle content.

For our discussion, the relevant terms in the UV completion are

$$-\mathcal{L}_{\text{int}}^{\text{II}} = Y_{\alpha\beta}^k \overline{\ell_{L\alpha}} \Delta_k^\dagger \ell_{L\beta}^c + \mu_{k,ab} \tilde{\phi}_a^T \Delta_k \tilde{\phi}_b + \dots, \quad (36)$$

which leads, after the decoupling of the heavy states of mass M_k , to the effective coupling

$$\frac{K_{\alpha\beta}^{ab}}{\Lambda} = -\frac{2\mu_{k,ab}}{M_k^2} Y_{\alpha\beta}^k. \quad (37)$$

An equivalent way is saying that the Higgs triplets acquire small vevs of the form

$$\langle \Delta_k^\dagger \rangle = -\frac{\mu_{k,ab}}{M_k^2} \langle \tilde{\phi}_a \rangle \langle \tilde{\phi}_b \rangle^T. \quad (38)$$

In order to extend the analysis previously done for the effective operator, we just need to replace the field combination $\tilde{\phi} \tilde{\phi}^T$ by Δ^\dagger . Thus, we require the flavour charge \mathcal{Q}_F , associated with the flavour group, to satisfy the relation $\mathcal{Q}_F(\Delta_k) = \mathcal{Q}_F(\phi_i) \mathcal{Q}_F(\phi_j)$, and the corresponding field transformation $\Delta_k \rightarrow \mathcal{Q}_F(\Delta_k) \Delta_k$. For each different i and j combination at the effective level, a triplet scalar Δ_k should be introduced. This means that effective models with the $\tilde{\phi}_1 \tilde{\phi}_2^T$ combination require three Higgs triplets to be implemented. These UV models will then contain an extended scalar sector with two Higgs doublets and three Higgs triplets (or two triplets in case (d) of the class $\tilde{\mathbf{B}}$ given in Eq. (25)). The field transformations are

$$\begin{aligned} \phi_1 &\rightarrow \phi_1, & \phi_2 &\rightarrow \omega_n \phi_2, \\ \Delta_1 &\rightarrow \Delta_1, & \Delta_2 &\rightarrow \omega_n^2 \Delta_2, & \Delta_3 &\rightarrow \omega_n \Delta_3. \end{aligned} \quad (39)$$

Since we have considerably enlarged the scalar sector, it is important to check whether accidental symmetries can be avoided in the scalar potential. For a model with two scalar doublets and several scalar triplets, the phase sensitive terms in the scalar potential are $\phi_a^\dagger \phi_b, (\phi_a^\dagger \phi_b)^2, \Delta_i^\dagger \Delta_j, (\Delta_i^\dagger \Delta_j)^2, \Delta_i^\dagger \Delta_j \Delta_k^\dagger \Delta_l, \phi_a^* \phi_b^* \Delta_k$, and $\phi_a^\dagger \phi_b \Delta_i^\dagger \Delta_j$.

A Z_n group is a symmetry of the scalar potential if it does not induce a larger symmetry. Therefore, one needs to check for terms that under the field transformations transform with ω_n^k . For example, the term $\phi_a^* \phi_b^* \Delta_k$, even though is phase sensitive, it is by construction invariant and, consequently, not sensitive to the order of the group. On the other hand, the term $\phi_1^\dagger \phi_2 \Delta_1^\dagger \Delta_2$ transforms with ω_n^3 . The presence of this term in the potential implies $n = 3$, that is, the group is Z_3 and not $U(1)$. It is easy

to check that the term with the largest phase transformation is $(\Delta_1^\dagger \Delta_2)^2$, which transforms with ω_n^4 . Therefore, the largest discrete Abelian symmetry allowed in a 2HDM plus three (or two) scalar triplets is Z_4 . From Table 2, we then conclude that cases leading to a Z_5 symmetry will have an accidental continuous symmetry in the scalar potential. Other cases, including the $Z_2 \times Z_3$ one, can have an UV completion in a type II seesaw framework without introducing continuous symmetries in the scalar potential.

Defining the field vector (ϕ_a, Δ_k) we can determine the D matrix for each case. Consider first the Z_8 case in class $\tilde{\mathbf{B}}$. Since the scalar phase transformation of ϕ_2 is $\omega_8^2 = \omega_4$, the scalar field transformations are given by

$$\begin{aligned} \phi_1 &\rightarrow \phi_1, & \phi_2 &\rightarrow \omega_4 \phi_2, \\ \Delta_1 &\rightarrow \Delta_1, & \Delta_2 &\rightarrow \omega_4^2 \Delta_2, & \Delta_3 &\rightarrow \omega_4 \Delta_3. \end{aligned} \quad (40)$$

The set of phase sensitive interactions yields the matrix

$$D = \begin{pmatrix} -2 & 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 & -1 \\ -1 & 1 & 0 & -1 & 1 \\ 0 & 0 & -2 & 2 & 0 \end{pmatrix}, \quad (41)$$

which leads to $D_{SNF} = \text{diag}(1, 1, 1, 4, 0)$, as expected. Therefore, this model has a Z_8 flavour symmetry in the full (scalar + leptonic) Lagrangian. Nevertheless, in the scalar potential only the Z_4 subgroup acts nontrivially.

Next let us consider the $Z_2 \times Z_2 \times Z_3$ case of class $\tilde{\mathbf{B}}$. The scalar field transformations are now given by

$$\begin{aligned} \phi_1 &\rightarrow (1, 1, 1) \phi_1, & \phi_2 &\rightarrow (1, -1, \omega) \phi_2 \\ \Delta_1 &\rightarrow (1, 1, 1) \Delta_1, & \Delta_2 &\rightarrow (1, 1, \omega^2) \Delta_2. \end{aligned} \quad (42)$$

The first Z_2 is irrelevant, since it simply reflects a discrete lepton number. Therefore, we focus on the possible implementation of $Z_2 \times Z_3$ in the scalar potential with two Higgs doublets and two Higgs triplets. As already pointed out, this implies an accidental symmetry in the scalar potential. There is, however, a way out in this case. Looking at the symmetry implementation in the effective operator, one sees that while Z_3 gives the flavour structure, the only purpose of Z_2 is to forbid the cross term $\phi_1^* \phi_2^*$, i.e. it is just a shaping symmetry. In our type II implementation, the above cross term has been avoided by removing the triplet scalar Δ_3 associated with it, and keeping only $\Delta_{1,2}$. Therefore, at the UV level we only have the Z_3 symmetry for the scalar fields, implying

$$D = \begin{pmatrix} -2 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ -1 & 1 & -1 & 1 \end{pmatrix}, \quad (43)$$

with the Smith normal form $D_{SNF} = \text{diag}(1, 1, 3, 0)$, as expected. The full Lagrangian has a $Z_2 \times Z_3$ symmetry, where the Z_2 corresponds to the discrete lepton number.

Table 3: Symmetries in the effective model and type II UV completion. The trivial Z_2 associated with lepton number has been omitted.

Texture	Effective 2HDM	2HDM + type II	Scalar potential	Goldstone boson
$\tilde{\mathbf{A}}$	$U(1)$	$U(1)$	$U(1)$	Yes
$\tilde{\mathbf{B}}$	Z_5	Z_5	$U(1)$	Pseudo
	Z_8	Z_8	Z_4	No
	$Z_2 \times Z_3$	Z_3	Z_3	No
$\tilde{\mathbf{C}}$	$U(1)$	Z_8	Z_4	No

After the decoupling of the heavy triplets, the effective Lagrangian sees its symmetry being enlarged.

Finally we consider class $\tilde{\mathbf{C}}$. Although the flavour symmetry is continuous in the effective approach, it may be accidental. As can be seen from Eq. (34), the effective texture can be implemented by a Z_8 symmetry. In the scalar sector, the field transformations are exactly the same as in case Z_8 of class $\tilde{\mathbf{B}}$. Therefore, for the scalar sector, D has the same form of Eq. (41). This implies that no accidental symmetries appear in the scalar sector. The model can then be implemented from a Z_8 flavour symmetry group. The Yukawa sector alone will still have an accidental global $U(1)$ symmetry, as can be checked by constructing the D matrix for that sector only. Yet the full Lagrangian and, most importantly, the scalar potential has only a discrete symmetry avoiding unwanted (pseudo-) Goldstone bosons.

We summarize our results in Table 3, where we present the allowed symmetries in the effective 2HDM and in the corresponding type II seesaw UV completion.

Some of the symmetry implementations presented in this section have been previously studied [5, 7]. However, the discrete symmetry groups differ in some of them. In Refs. [5, 7], textures $\mathbf{A}_{1,2}$ and $\mathbf{B}_{3,4}$, which in our case belong to class $\tilde{\mathbf{A}}$, are implemented with Z_6 . As we have shown, any discrete group Z_k , with $k \geq 5$, implies a global accidental $U(1)$ in the full Lagrangian. The textures $\mathbf{B}_{1,2}$ are implemented in the above works using two Higgs triplets and a Z_3 symmetry. This is precisely our last case of class $\tilde{\mathbf{B}}$. There are, however, two other minimal implementations of these zero textures, which are given in Table 3. Concerning the implementation of texture \mathbf{C} , in the above works a Z_4 discrete symmetry is used, but the scalar Higgs doublets were ignored. As shown before, while Z_4 is the symmetry of the scalar potential, the full Lagrangian requires a Z_8 flavour group. We also note that in all the implementations of Refs. [5, 7] the flavour sym-

metry needs to be softly broken in the scalar potential. This is due to the fact that only one Higgs doublet is introduced in the theory. It is precisely the existence of such soft-breaking terms that permits the implementation of \mathbf{C} with a Z_4 and not a Z_8 symmetry.

A final remark on the Type-II UV completions is in order. We have enlarged the scalar sector in such a way that, in some cases, it is enough to have no accidental continuous symmetries in the scalar potential. These extensions may solve the global $U(1)$ scalar problem, but they may have difficulties with the presence of very light scalar particles. In order to see this, let us take as an example the Z_8 case in class $\tilde{\mathbf{B}}$. There is a single term that requires a Z_4 symmetry in the scalar potential, namely, $\lambda_\Delta (\Delta_1^\dagger \Delta_2)^2$. In the limit $\lambda_\Delta \rightarrow 0$, we recover the $U(1)$ symmetry of the effective potential. This term appears in the scalar mass matrix diagonalization, once ϕ_a and Δ_i acquire vevs. However, $\langle \Delta_i \rangle$ should be very small in order to explain the neutrino masses. This implies that the corrections to the massless scalars are also very small.

6.2. Type-I seesaw realization

In the canonical type I seesaw scenario, three right-handed neutrinos, N_{iR} , with heavy masses in order to explain the light neutrino masses, are added to the SM particle content. The UV Lagrangian is of the form

$$-\mathcal{L}_{\text{int}}^{\text{I}} = Y_{\alpha i}^a \overline{\ell_{L\alpha}} \tilde{\phi}_a N_{iR} + \frac{1}{2} M_{ij} \overline{N_{iR}^c} N_{jR}, \quad (44)$$

which leads, after the decoupling of the heavy states, to the effective coupling

$$\frac{K_{\alpha\beta}^{ab}}{\Lambda} = -Y^a M^{-1} Y^{bT}. \quad (45)$$

Contrarily to the type II seesaw, in this UV completion the effective coupling is not directly extracted from the UV Lagrangian, which usually makes the construction of these models more challenging. In order to obtain the possible implementations, we recall that the generator \mathcal{S}_L has no degeneracy in all possible effective implementations. It remains to find the way that the heavy right-handed neutrino fields transform under the flavour symmetry. Up to permutations, we can split the analysis into three cases: $\mathcal{S}_R = \pm \mathbb{1}$, $\mathcal{S}_R = \pm \text{diag}(1, 1, -1)$, and $\mathcal{S}_R = \text{diag}(\pm 1, e^{i\beta}, e^{-i\beta})$ with $\beta \neq \pi$.

In the first case, the matrix M is completely general (texture A_1), and the Yukawa textures contain just a line of nonzero entries. One can easily check that $Y^a M^{-1} Y^{aT} \sim Y^a Y^{aT}$ has a diagonal texture (some diagonal entries may be zero). Since none of the realizable cases has two textures of the diagonal form, this scenario is excluded.

In the second case, the matrix M (and its inverse) has a block-diagonal form (texture A_2). Once again, it turns out that $Y^a M^{-1} Y^{aT} \sim Y^a Y^{aT}$ has a diagonal texture. Therefore, this case is also excluded.

There remains the case with $\mathcal{S}_R = \text{diag}(\pm 1, e^{i\beta}, e^{-i\beta})$. In this case, the right-handed neutrino mass matrix is given by

$$M = \begin{pmatrix} \times & & \\ & \times & \\ & & \times \end{pmatrix} = P_{23} A_{13}. \quad (46)$$

Since all the phase transformations are known, each possibility can be straightforwardly analysed. For class $\tilde{\mathbf{A}}$, the only possible implementation up to permutations on the right side is given by Eq. (31), with the additional field transformations

$$Z_2 \times U(1) : \begin{cases} N_1 \rightarrow (-1, 1) N_1, & N_2 \rightarrow (1, e^{i\alpha}) N_2, \\ N_3 \rightarrow (1, e^{-i\alpha}) N_3. \end{cases} \quad (47)$$

Note that $U(1)$ is the group that completely defines the flavour structure of the couplings, while the additional Z_2 reflects an accidental symmetry when that structure is present. At the UV level, the fields $\ell_{\alpha L}$ and ϕ_a will transform exactly as in Eq. (31) under the $U(1)$ group. However, under the Z_2 symmetry their transformation is now given by

$$Z_2 : \ell_{\mu L} \rightarrow -\ell_{\mu L}, \quad \phi_2 \rightarrow -\phi_2, \quad (48)$$

with the remaining fields invariant. The Yukawa couplings are given by

$$Y_1^\nu = \begin{pmatrix} \times & & \\ \times & & \\ & \times & \end{pmatrix}, \quad Y_2^\nu = \begin{pmatrix} & \times & \\ & & \times \\ \times & & \end{pmatrix}. \quad (49)$$

The minimal discrete implementation is then obtained with the replacement $e^{i\alpha} \rightarrow \omega_5$.

It is instructive to make the connection between the Z_2 symmetry at high and low energies. Looking at the way $\ell_{\alpha L}$ and ϕ_a transform at high energies, it is not evident that this symmetry corresponds to the discrete lepton number at low energies. To check this, let us decouple the right-handed fields and write the effective Lagrangian as

$$-\mathcal{L}_{\text{int}} \sim (\overline{\ell_{eL}} \ell_{\tau L}^c + \overline{\ell_{\tau L}} \ell_{eL}^c + \overline{\ell_{\mu L}} \ell_{\mu L}^c) \tilde{\phi}_1 \tilde{\phi}_1^T + (\overline{\ell_{\mu L}} \ell_{\tau L}^c + \overline{\ell_{\tau L}} \ell_{\mu L}^c) \tilde{\phi}_1 \tilde{\phi}_2^T + \overline{\ell_{\tau L}} \ell_{\tau L}^c \tilde{\phi}_2 \tilde{\phi}_2^T, \quad (50)$$

where, for simplicity, we have omitted the coupling in each term. The terms of type ϕ_a^2 are insensitive to the change $\ell_{\mu L} \rightarrow -\ell_{\mu L}$ under Z_2 . In the cross term, this field transformation is indistinguishable from demanding $\phi_1 \rightarrow -\phi_1$. The question is whether the latter replacement leads to any change in the other Lagrangian terms. If ϕ_1 couples to the charged lepton Yukawa term, then the $e_{\beta R}$ charges can be properly chosen to account for this transformation. Concerning the scalar potential, since it has a $U(1)$ global symmetry the only cross term is $|\phi_1^\dagger \phi_2|^2$, which is insensitive to the above transformation. Therefore, at low energies the Z_2 symmetry can be expressed as in Eq. (31).

In the case of class $\tilde{\mathbf{B}}$, there are only discrete groups. Checking all possible charge assignments to N_{iR} under the flavour group, we found no viable implementation within this class.

Finally, for class $\tilde{\mathbf{C}}$ the only possible implementation up to permutations on the right is given by Eq. (34) with the additional field transformations

$$Z_2 \times U(1) : \begin{cases} N_1 \rightarrow (-1, 1)N_1, & N_2 \rightarrow (1, e^{i\alpha/2})N_2, \\ N_3 \rightarrow (1, e^{-i\alpha/2})N_3. \end{cases} \quad (51)$$

The Yukawa textures take the form

$$Y_1^\nu = \begin{pmatrix} & \times & \\ \times & & \\ & & \end{pmatrix}, \quad Y_2^\nu = \begin{pmatrix} & \times & \\ & & \\ & & \times \end{pmatrix}. \quad (52)$$

The discrete implementation is obtained with the replacement $e^{i\alpha/2} \rightarrow \omega_8$.

The results for all cases are summarized in Table 4, where the allowed symmetries in the effective 2HDM and the corresponding type I seesaw UV completions are given.⁴

Table 4: Symmetries in the effective model and type I UV completion. The trivial Z_2 associated with lepton number has been omitted.

Texture	Effective 2HDM	2HDM + type I
$\tilde{\mathbf{A}}$	$U(1)$	$Z_2 \times U(1)$
$\tilde{\mathbf{B}}$	Z_5 Z_8 $Z_2 \times Z_3$	Non-realizable
$\tilde{\mathbf{C}}$	$U(1)$	$Z_2 \times U(1)$

7. Conclusions

We have obtained the minimal Abelian symmetry realizations of phenomenological two-zero neutrino textures, i.e. neutrino mass matrices with two zeros, written in the physical basis where the charged leptons are diagonal. The symmetry constructions were achieved resorting to the canonical and Smith normal form methods. The implementation of these symmetries in UV completions based on the type I and type II seesaw mechanisms was

⁴A symmetry realization of the phenomenologically viable FGM two-zero texture neutrino mass matrices within the framework of the mixed type-I + type-II seesaw mechanism has been considered in Ref. [15].

also presented. It is worth noticing that the discrete symmetry realizations of the two-zero neutrino textures presented here are different from previous studies [4, 5, 7]. Indeed, in our implementations the flavour symmetry in the leptonic sector is only broken at the electroweak scale and not at the (high) seesaw scale. This means, in particular, that the texture zeros in the effective neutrino mass matrix will remain exact up to the electroweak scale, without being affected by renormalization group corrections.

Finally, we also remark that the minimal effective and seesaw-like implementations of the neutrino textures typically suffer from the presence of very light (or even massless) scalars. This is due to the existence of only two Higgs doublets at the electroweak scale, transforming under an Abelian group of order greater than two. In this work, we have focused on the leptonic sector only; it may happen that quarks interact with other scalar doublets or that additional scalar doublets, associated with repeated textures, appear in the neutrino sector. In such cases, larger symmetries of the scalar potential are allowed. Independently of the implementation chosen, if we insist on curing this problem without breaking softly the flavour symmetry, we need to extend the 2HDM scalar potential at the electroweak scale.

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Appendix A. Discrete symmetries, texture decompositions and charges

In this appendix we construct the minimal discrete implementations of the allowed textures, and the corresponding phase transformation matrices, using the canonical method [12]. In order to exemplify the procedure we consider the simple case when $\mathcal{S}_L = \text{diag}(1, 1, e^{i\alpha})$. We start by replacing the continuous phase by a discrete one, i.e. $e^{i\alpha} \rightarrow \omega_n^k$, where the index n is the order of the group and k is the integer associated charge. The phase transformation matrix Θ is a matrix containing in its entries the associated phase transformation of the lepton fields. Therefore, for a term of the form $L^* L^* \Sigma$, with Σ a scalar field, we get

$$\Theta = \begin{pmatrix} 1 & 1 & \omega_n^{-k} \\ 1 & 1 & \omega_n^{-k} \\ \omega_n^{-k} & \omega_n^{-k} & \omega_n^{-2k} \end{pmatrix}. \quad (\text{A.1})$$

From this matrix we obtain two different implementations: $\omega_n^{-2k} = 1$ or $\omega_n^{-2k} \neq 1$.

Table A.1: Discrete symmetry, two-zero texture decomposition and Higgs combination charges obtained from the canonical method. Here $P = \{1, P_{12}, P_{13}, P_{23}, P_{123}, P_{321}\}$, $P' = \{1, P_{12}, P_{13}\}$ and $P'' = \{1, P_{13}, P_{23}\}$.

Symmetry	Texture decomposition	Charges
None	A_1	
Z_{2n}	$P \{A_2 \oplus A_3\} P$	$(1, \omega_{2n}^n); k = n$
$Z_{n \geq 3}$	$P \{A_7 \oplus A_3 \oplus A_{12}\} P$	$(1, \omega_n^k, \omega_n^{2k})$
Z_{3n}	$A_{13}P_{23} \oplus A_{13}P_{12} \oplus A_{13}P_{13}$	$(1, \omega_{3n}^n, \omega_{3n}^{2n}); k_1 = -k_2 = n$
Z_{4n}	$P' \{A_{13}P_{23} \oplus P_{13}A_{15}P_{123} \oplus A_{15} \oplus P_{123}A_{15}P_{12}\} P'$	$(1, \omega_{4n}^n, \omega_{4n}^{2n}, \omega_{4n}^{3n}); k_1 = -k_2 = n$
$Z_{n \geq 5}$	$P' \{A_{13}P_{23} \oplus P_{13}A_{15}P_{123} \oplus P_{123}A_{15}P_{12} \oplus P_{23}A_{12}P_{23} \oplus A_{12}\} P'$	$(1, \omega_n^{k_1}, \omega_n^{-k_1}, \omega_n^{2k_1}, \omega_n^{-2k_1}); k_2 = -k_1$
$Z_{2(n+2)}$	$P'' \{P_{13}A_{15}P_{13} \oplus P_{13}A_{15}P_{123} \oplus P_{123}A_{15}P_{12} \oplus A_{15}P_{23} \oplus A_{12} \oplus A_0\} P''$	$(1, \omega_{2(n+2)}^{n+2}, \omega_{2(n+2)}^{k_2}, \omega_{2(n+2)}^{k_2+n+2}, \omega_{2(n+2)}^{2k_2}); k_1 = n+2$
$Z_{n \geq 6}$	$P_{13}A_{12}P_{13} \oplus P_{13}A_{15}P_{123} \oplus P_{123}A_{15}P_{12} \oplus A_{15}P_{23} \oplus P_{23}A_{12}P_{23} \oplus A_{12}$	$(1, \omega_n^{k_1}, \omega_n^{k_2}, \omega_n^{k_2+k_1}, \omega_n^{2k_1}, \omega_n^{2k_2})$
$Z_{2n} \times Z_{2m}$	$P \{A_{13} \oplus P_{321}A_{15}P_{13} \oplus P_{123}A_{15}P_{12} \oplus A_{15}P_{23}\} P$	$[(1, 1), (1, \omega_{2m}^m), (\omega_{2n}^n, 1), (\omega_{2n}^n, \omega_{2m}^m)]; k = n, k' = m$

If $\omega_n^{-2k} = 1$, then $2k = 0 \pmod{n}$ and the order of the group has to be a multiple of two. We define the group as Z_{2n} . The implementation, or chain, is then given by $A_2 \oplus A_3$ with the associated charges $(1, \omega_{2n}^n)$. When $\omega_n^{-2k} \neq 1$, $k \neq 2k \neq 0$ and the order of the group has to be $n \geq 3$. We define the group as $Z_{n \geq 3}$. The corresponding chain is given by $A_7 \oplus A_3 \oplus A_{12}$, with the charges $(1, \omega_n^k, \omega_n^{2k})$.

The above procedure can be repeated for the remaining cases. For the cases where there is no degeneracy, the discrete form of the left generator is given by $\mathcal{S}_L = \text{diag}(1, \omega_n^{k_1}, \omega_n^{k_2})$. The results are displayed in Table A.1.

Note that the vector of charges indicates the way that the auxiliary field Σ transforms in order to pick a given texture. For instance, in the type II seesaw, the field Σ is identified with the scalar triplet Δ^\dagger , while in the general dimension-5 effective approach it is identified with the Higgs doublet combination $\phi_a^* \phi_b^*$.

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